Standard Model	**Sheet 2**	25.02.2019

Exercise 1: Constructing propagators. Consider the QED Lagrangian $\mathscr{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{2\xi}(\partial_\mu A^\mu)^2$, where ξ is a gauge parameter. Let us write down the action ($S = \int_X \mathscr{L}$) in Fourier representation, so that the quadratic part ($=: S_0$) becomes

$$iS_0 = -\int_{P,Q} \Big\{ \frac{1}{2} A_\alpha(P)\delta(P+Q)\, M_1^{\alpha\beta}(P) A_\beta(Q) + \bar{\psi}_\rho(P)\delta(P-Q)\, M_2^{\rho\sigma}(P)\psi_\sigma(Q) \Big\} ,$$

where α, β are Lorentz indices and ρ, σ are spinor indices.

(a) Construct the inverse matrices M_1^{-1}, M_2^{-1}.

(b) Suppose that we add a term $\propto \epsilon_{\alpha\beta\mu\nu} F^{\alpha\beta} F^{\mu\nu}$ to the Lagrangian, where $\epsilon_{\alpha\beta\mu\nu}$ is a totally antisymmetric tensor, with $\epsilon_{0123} := 1$. Show that this would not change M_1^{-1}.

Exercise 2: Time ordering. Let us fix the spatial momentum $\vec{p}$ and consider two "scalar" propagators with different prescriptions for circumventing the poles on the integration contour:

$$\Delta_T(t,t') := \int_{-\infty}^{\infty} \frac{\mathrm{d}P_0}{2\pi} \frac{i\, e^{-iP_0(t-t')}}{P_0^2 - E_p^2 + i0^+} , \quad \Delta_R(t,t') := \int_{-\infty}^{\infty} \frac{\mathrm{d}P_0}{2\pi} \frac{i\, e^{-iP_0(t-t')}}{(P_0 + i0^+)^2 - E_p^2} .$$

(a) Argue from the integral representations that both functions separately satisfy the *inhomogeneous* equation $(\partial_t^2 + E_p^2)\Delta_{T,R} = -i\delta(t-t')$.

(b) Sketch the poles in the complex plane and, using the residue theorem to compute $\Delta_T - \Delta_R$, show that it satisfies the *homogeneous* equation $(\partial_t^2 + E_p^2)(\Delta_T - \Delta_R) = 0$.

(c) Verify that the difference between $\Delta_{T,R}$ lies in the boundary conditions: Δ_T is "time-ordered", $\Delta_T(t,t') = \Delta_T(t',t)$, whereas Δ_R is "retarded", $\Delta_R(t,t') = 0$ for $t < t'$.

Exercise 3: QED processes. Draw the "tree-level" (i.e. no closed loops) QED Feynman diagrams for the following processes:

(a) $e^- + \mu^- \to e^- + \mu^-$ ("Mott scattering").

(b) $e^- + e^- \to e^- + e^-$ ("Møller scattering").

(c) $e^- + e^+ \to e^- + e^+$ ("Bhabha scattering").

(d) $e^- + \gamma \to e^- + \gamma$ ("Compton scattering").

(e) $e^- + e^+ \to \gamma + \gamma$ ("pair annihilation").

(f) $\gamma + \gamma \to e^- + e^+$ ("pair creation").

Exercise 4: Invariant amplitude. Consider a theory with three different real scalar particles, interacting through the Lagrangian

$$\mathscr{L}_I := g\, \phi_A\, \phi_B\, \phi_C .$$

The masses satisfy $M_A > M_B + M_C$, so that the decay $A \to B + C$ is kinematically allowed.

(a) Draw the Feynman diagrams of orders $\mathscr{O}(g)$, $\mathscr{O}(g^2)$, $\mathscr{O}(g^3)$ for this process.

(b) Write down the absolute value of the amplitude, $|\mathscr{M}|$, at order $\mathscr{O}(g^3)$.