Standard Model	**Sheet 1**	18.02.2019

Exercise 1: Relativistic kinematics. Consider the reaction $p + p \to p + p + p + \bar{p}$.

(a) In a "fixed target" experiment, one of the initial protons is at rest. How much energy should the other proton have in order for the process to be kinematically allowed?

(b) At the Large Hadron Collider (LHC), the protons collide head-on with the same velocity. What is the energy threshold in this case?

Exercise 2: Phase space. The phase space integration measure is defined as

$$\mathrm{d}\Phi_n := \prod_{i=1}^{n} \frac{\mathrm{d}^3\vec{p}_i}{(2\pi)^3 2E_{p_i}} (2\pi)^4 \delta^{(4)}(\sum_{j=1}^{m} K_j - \sum_{i=1}^{n} P_i) ,$$

whereby K_j are the four-momenta in the initial state, P_i are those in the final state, and $(P_i)^0 := E_{p_i} := \sqrt{m_i^2 + p_i^2}$, where $p_i \equiv |\vec{p}_i|$. Show that this measure is Lorentz-invariant.

Exercise 3: Two-particle decay. Consider two-particle decay in the rest frame of the decaying particle:

$$\Gamma = \frac{1}{2M} \int \mathrm{d}\Phi_2 \, |\mathscr{M}|^2(\vec{p}_1, \vec{p}_2) \, .$$

The decaying particle has the mass M, the decay products the masses m_1 and m_2. Let us assume that $|\mathscr{M}|^2(\vec{p}_1, -\vec{p}_1)$ is only a function of $p_1 := |\vec{p}_1|$, i.e. $|\mathscr{M}|^2(\vec{p}_1, -\vec{p}_1) \to |\mathscr{M}|^2(p_1)$.

(a) Show that

$$\Gamma = \frac{\rho_0}{8\pi M^2} \; |\mathscr{M}|^2(\rho_0) \, \theta(M - m_1 - m_2) \, ,$$

with

$$\rho_0 = \frac{1}{2M} \sqrt{M^4 + m_1^4 + m_2^4 - 2M^2 m_1^2 - 2M^2 m_2^2 - 2m_1^2 m_2^2} \, .$$

(b) What is the physical meaning of ρ_0?

(c) Show that a massive particle cannot radiate a photon.

Exercise 4: Three-particle decay. Consider a situation like in Exercise 3 but with three particles in the final state, with masses m_1, m_2 and m_3. Show that, unlike in two-particle decay, p_1 is no longer a uniquely fixed quantity. Can you determine max(p_1)? [Note: This kinematics lead Pauli to postulate the existence of neutrinos in 1930.]