4.2 Anomalies

As we have seen, the Standard Model Lagrangian contains two types of symmetries: exact gauge symmetries, and exact or approximate global symmetries.

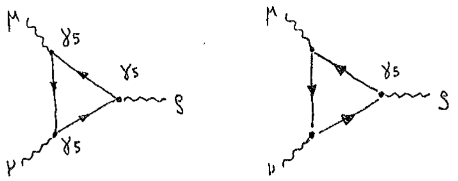
When the theory is quantized through path integrals, it is no longer sufficient to only inspect the invariances of the Lagrangian. The <u>integration measure</u> should also remain invariant; if this is not the case, we speak of an <u>anomaly</u>.

If an anomaly exists, the Lagrangian symmetry is an illusion; in other words, it is „explicitely" broken in the quantum theory. Were this the case, all consequences of symmetries would also be lost:

(i) no gauge symmetry $\Rightarrow$ no proven renormalizability $\Rightarrow$ disaster!

(ii) no global symmetry $\Rightarrow$ would-be Goldstone boson or Noether current is absent $\Rightarrow$ interesting physics!

In the Standard Model, the most prominent anomaly, known as the <u>triangle anomaly</u> or <u>Adler-Bell-Jackiw anomaly</u> or <u>chiral anomaly</u>, is related to <u>fermions</u> which couple to gauge fields <u>via γ_5</u>:



($\gamma_5 := \gamma^5$ from p. 13)

We immediately observe that there is <u>no triangle gauge anomaly in QCD</u>, since the theory is vectorlike (p. 41). Nevertheless, as we will see, colour degrees of freedom play a role in anomalies related to $SU_L(2)\times U_Y(1)$.

For $SU_L(2)\times U_Y(1)$: (cf. p. 27; p. 28; Exercise 7.3)

$$D_\mu L = \partial_\mu L - i\left[\left(g_w T^a A^a_\mu + g_Y T^0 B_\mu\right)\left(\tfrac{1-\gamma_5}{2}\right) + g_Y\begin{pmatrix}0&0\\0&1\end{pmatrix}\left(\tfrac{1+\gamma_5}{2}\right)\right]\begin{pmatrix}\nu\\ e\end{pmatrix}$$

$$= \dots - i\Big[g_w \overset{+\frac12}{T^3_{11}} A^3_\mu + g_Y \overset{+\frac12}{Y_{\nu L}} B_\mu\Big]\left(\tfrac{1-\gamma_5}{2}\right)\nu - i\Big[g_Y \overset{0}{Y_{\nu R}} B_\mu\Big]\left(\tfrac{1+\gamma_5}{2}\right)\nu$$

$$- i\Big[g_w \underset{-\frac12}{T^3_{22}} A^3_\mu + g_Y \underset{+\frac12}{Y_{eL}} B_\mu\Big]\left(\tfrac{1-\gamma_5}{2}\right)e - i\Big[g_Y \underset{+1}{Y_{eR}} B_\mu\Big]\left(\tfrac{1+\gamma_5}{2}\right)e$$

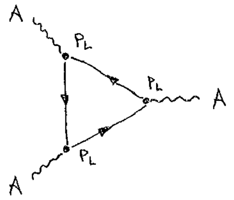
focus on neutral currents

$$D_\mu Q_\alpha = \dots - i\Big[g_w \overset{+\frac12}{T^3_{11}} A^3_\mu + g_Y \overset{-\frac16}{Y_{uL}} B_\mu\Big]\left(\tfrac{1-\gamma_5}{2}\right)u_\alpha - i\Big[g_Y \overset{-\frac23}{Y_{uR}} B_\mu\Big]\left(\tfrac{1+\gamma_5}{2}\right)u_\alpha$$

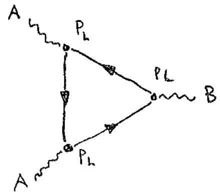
$$- i\Big[g_w \underset{-\frac12}{T^3_{22}} A^3_\mu + g_Y \underset{-\frac16}{Y_{dL}} B_\mu\Big]\underset{P_L}{\left(\tfrac{1-\gamma_5}{2}\right)}d_\alpha - i\Big[g_Y \underset{+\frac13}{Y_{dR}} B_\mu\Big]\underset{P_R}{\left(\tfrac{1+\gamma_5}{2}\right)}d_\alpha$$

Cancellation of gauge anomalies

Let us inspect terms of the types A^3, A^2B, AB^2, B^3 in turn. Since $P_L P_R = 0$, all vertices need to have the same chiral structure.

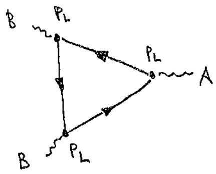


$$\propto \Big[\underbrace{(T^3_{11})^3}_{\text{from up-type}} + \underbrace{(T^3_{22})^3}_{\text{from down-type}} \Big]\big(\underbrace{1}_{\text{leptons}} + \underbrace{3}_{\text{quarks}} \big) = \Big[\tfrac{1}{8} - \tfrac{1}{8}\Big](1+3) \overset{!}{=} 0.$$



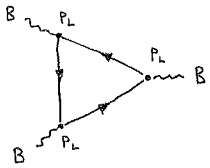
$$\propto (T^3_{11})^2\, Y_{\nu L} + (T^3_{22})^2\, Y_{eL} + 3(T^3_{11})^2\, Y_{uL} + 3(T^3_{22})^2\, Y_{dL}$$

$$= \frac{1}{4}\Big[\frac{1}{2} + \frac{1}{2} + 3\Big(-\frac{1}{6}\Big) + 3\Big(-\frac{1}{6}\Big) \Big] \overset{!}{=} 0.$$



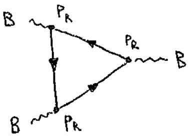
$$\propto (T^3_{11})\big[Y_{\nu L}^2 + 3Y_{uL}^2 \big] + (T^3_{22})\big[Y_{eL}^2 + 3Y_{dL}^2 \big]$$

$$= \frac{1}{2}\Big[\frac{1}{4} + 3\cdot\Big(-\frac{1}{6}\Big)^2 \Big] - \frac{1}{2}\Big[\frac{1}{4} + 3\Big(-\frac{1}{6}\Big)^2 \Big] \overset{!}{=} 0.$$



$$\propto Y_{\nu L}^3 + Y_{eL}^3 + 3\big[Y_{uL}^3 + Y_{dL}^3 \big]$$

$$= \frac{1}{8} + \frac{1}{8} + 3\Big[\Big(-\frac{1}{6}\Big)^3 + \Big(-\frac{1}{6}\Big)^3 \Big] = \frac{1}{4} - \frac{1}{36} \neq 0.$$



$$\propto Y_{\nu R}^3 + Y_{eR}^3 + 3\big[Y_{uR}^3 + Y_{dR}^3 \big]$$

$$= 1 + 3\Big[\Big(-\frac{2}{3}\Big)^3 + \Big(\frac{1}{3}\Big)^3 \Big] = 1 + \frac{-8+1}{9} \neq 0.$$

But note: $P_L^3 = \frac{1}{8}\big(1 \overset{!}{-} 3\gamma_5 + 3\gamma_5^2 \overset{!}{-} \gamma_5^3\big)$; $P_R^3 = \frac{1}{8}\big(1 \overset{!}{+} 3\gamma_5 + 3\gamma_5^2 \overset{!}{+} \gamma_5^3\big)$.

So for the dangerous terms we need to subtract the two results:

$$\frac{9-1}{36} - \frac{9-8+1}{9} = \frac{8}{36} - \frac{8}{36} \overset{!}{=} 0.$$

(Remark: In addition to the triangle anomaly, there is a so-called „Witten's global SU(2) anomaly", related to topological properties of $SU_L(2)$ gauge fields. It is absent if the number of $SU_L(2)$ doublets is even. Since each generation has 1 lepton doublet and $N_c = 3$ quark doublets, this is indeed the case in the Standard Model.)

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Existence of global anomalies

Global anomalies have two important (and related) manifestations in QCD:

(i) If $\det \mathcal{M} = |\det \mathcal{M}| e^{i\theta}$, and we rotate θ away through an axial rotation (cf. p. 36), then a term

$$\mathcal{L}_{\text{anomaly}} \propto \theta \, \frac{g_s^2}{64\pi^2} \, \varepsilon^{\mu\nu\rho\sigma} H^A_{\mu\nu} H^A_{\rho\sigma}$$

is induced in the QCD Lagrangian (p. 41).

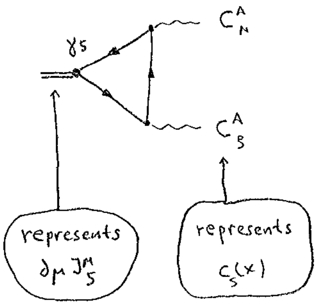
(ii) If we define a „Noether current" related to the axial rotation, $J^\mu_5 := \bar{q}\gamma^\mu\gamma_5 q$, then in a fixed gauge field background the current is not conserved (even in the massless limit):

$$\partial_\mu J^\mu_5 \propto 2 N_f \cdot \frac{g_s^2}{64\pi^2} \, \varepsilon^{\mu\nu\rho\sigma} H^A_{\mu\nu} H^A_{\rho\sigma} \, .$$

Some of the peculiar properties of the „topological charge density" here, $c_5(x) := \frac{g_s^2}{64\pi^2} \varepsilon^{\mu\nu\rho\sigma} H^A_{\mu\nu} H^A_{\rho\sigma}$, are discussed in Exercise 12.1.

Remarks:

* The phenomena above are again a consequence of triangle diagrams, but this time interpreted in another way:



$$\propto \sum_f 1^3 = N_f \, .$$

(If one considers a „non-singlet" axial current, $\tilde{J}^\mu_5 = \bar{q}\gamma^\mu\gamma_5\tilde{T}q$, with $\mathrm{Tr}[\tilde{T}] = 0$, then there is no anomaly: $\sum_f \tilde{T}_{ff} \cdot 1^2 = 0$.)

* Not only does the global anomaly appear with $SU_c(3)$, but it is in one respect more important there than with $SU_L(2) \times U_Y(1)$. Indeed, a possible term

$$\mathcal{L}_{\text{anomaly}} \propto \theta' \, \frac{g_w^2}{64\pi^2} \, \varepsilon^{\mu\nu\rho\sigma} \underbrace{G^a_{\mu\nu}}_{} G^a_{\rho\sigma} \quad \text{($SU_L(2)$ field strength)}$$

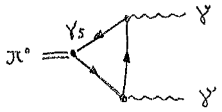
could be rotated away without changing $\det\mathcal{M}$, given that we are free to rotate right-handed fields freely, to compensate for the phase factor. (do not couple to $SU_L(2)$)

(As far as $U_Y(1)$ is concerned, a term $\propto \theta'' \frac{g_Y^2}{64\pi^2} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$ plays no role for the reason discussed in Exercise 12.1; that naive argument only fails for non-Abelian fields.)

Significance of global anomalies

Global anomalies have several interesting physics consequences:

(i) The important process $\pi^0 \to \gamma\gamma$ (99% of neutral pions decay this way) is mediated by the triangle diagram:



[J. Steinberger, Phys. Rev. 76 (1949) 1180]

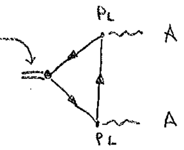
(ii) Whereas the $SU_A(N_f)$ subgroup of chiral symmetry is spontaneously broken, and leads to the existence of $N_f^2-1 = 8$ pseudo-Goldstone bosons (cf. p. 42-43), the subgroup $U_A(1)$ is explicitly broken by the anomaly. The corresponding particle, called η', is therefore heavy.

[E. Witten, Nucl. Phys. B 156 (1979) 269;
G. Veneziano, Nucl. Phys. B 159 (1979) 213]

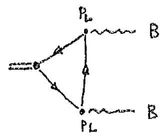
$N_g = 3$ stands for the number of generations

(iii) The vector current $J^\mu = \sum_{k=1}^{N_g} \{ \bar{Q}_k \gamma^\mu Q_k + \bar{L}_k \gamma^\mu L_k \}$ is not conserved in the presence of $SU_L(2) \times U_Y(1)$ gauge fields:

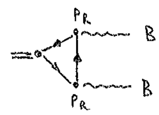
represents $\partial_\mu J^\mu$



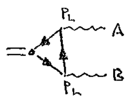
$$\propto \left[(T^3_{11})^2 + (T^3_{22})^2 \right](1+3) = \frac{1}{2} \times 4 = 2$$



$$\propto \underbrace{Y_{\nu L}^2 + Y_{eL}^2}_{\frac{1}{2}} + 3(\underbrace{Y_{uL}^2 + Y_{dL}^2}_{-\frac{1}{6}}) = \frac{1}{2} + 3 \times \frac{1}{18}$$



$$\propto Y_{\nu R}^2 + Y_{eR}^2 + 3(Y_{uR}^2 + Y_{dR}^2) = 1 + 3 \times \frac{5}{9}$$
$$\quad\; 0 \qquad\; 1 \qquad\;\; -\tfrac{2}{3} \quad\; \tfrac{1}{3}$$



Can be seen to vanish.

But the last diagram needs to be subtracted, because $P_L^2 = \frac{1}{4}(1 - 2\gamma_5 + \gamma_5^2)$, $P_R^2 = \frac{1}{4}(1 + 2\gamma_5 + \gamma_5^2)$. So,

$$\partial_\mu J^\mu \propto 2 N_g \frac{g_w^2}{64\pi^2} \varepsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}$$
$$+ \underbrace{\left(\frac{1}{2} + \frac{1}{6} - 1 - \frac{5}{3}\right)}_{-2} N_g \frac{g_Y^2}{64\pi^2} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} .$$

[G. 't Hooft, Phys. Rev. Lett. 37 (1976) 8]

We say that baryon + lepton number (B+L) is not conserved in the Standard Model.