4. Strong interactions

4.1 Lagrangian, symmetries

Jumping over many interesting historical developments, we briefly state the Lagrangian of Quantum Chromodynamics (QCD) in this section, and outline some of the important phenomena that it captures.

Starting point: From the point of view of QED (sec. 2) and weak interactions (sec. 3), quarks and leptons are alike; in Nature, they are not. We solve this problem by assigning to each quark 3 components and postulating the existence of a corresponding gauge symmetry, $SU_C(3)$. Like QED, QCD is vectorlike, i.e. does not break parity.

Lagrangian: Denote

$$q := \begin{pmatrix} u \\ d \\ s \\ c \\ b \\ t \end{pmatrix}, \quad u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}, \quad u_1 = \begin{pmatrix} u_{1,1} \\ u_{1,2} \\ u_{1,3} \\ u_{1,4} \end{pmatrix}.$$

$N_f = 6$ flavours

Each flavour has $N_c = 3$ colours.

Each colour is a four-component Dirac spinor.

Then $$\mathcal{L}_{QCD} = -\frac{1}{4} H^A_{\mu\nu} H^{A\mu\nu} + \bar{q}\,(i\gamma^\mu D_\mu - \mathcal{M})\,q + \mathcal{L}_{anomaly}.$$

Here: $$D_\mu = \partial_\mu - i g_s T^A C^A_\mu \; ;$$

g_s: Strong coupling

T^A: Generators of SU(3)

C^A_μ: Gluon fields

$\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s, m_c, m_b, m_t)$;

$H^A_{\mu\nu} = \partial_\mu C^A_\nu - \partial_\nu C^A_\mu + g_s f^{ABD} C^B_\mu C^D_\nu$,

where the „structure constants" satisfy $[T^A, T^B] = i f^{ABD} T^D$; and $A, B, D \in \{1, \ldots, 8\}$.

Remarks:

* The coupling g_s is larger than g_w, g_Y (cf. p. 27), so for QCD phenomena, we can usually set $g_w, g_Y \to 0$.

* The part $-\frac{1}{4} H^A_{\mu\nu} H^{A\mu\nu}$ has the same gauge-invariant structure as with photons (p. 5) and with $W^\pm, Z^0$. In contrast to photons, however, gluons interact with each other, because of the non-linear terms in $H^A_{\mu\nu}$.

* The part $\mathcal{L}_{anomaly}$ is discussed in sec. 4.2. It could be removed, however, by rewriting the mass term with a complex phase (cf. p. 36):

$$-\bar{q}\mathcal{M}q \;\rightarrow\; -\bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M}^\dagger q_L .$$

Symmetries:

* $\mathcal{L}_{QCD}$ respects $SU_c(3)$ gauge symmetry, because the covariant derivative has the proper structure (cf. Exercise 7.1). Quarks are chosen to lie in its „fundamental" representation.

* In addition there is a large global symmetry, called „chiral symmetry", whose form depends on the mass matrix.

Chiral symmetry:

* Similarly to p. 37, we can write

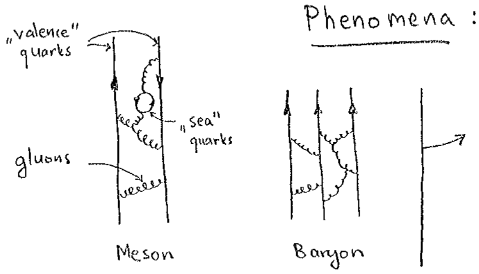
$$\delta \mathcal{L}_{QCD} = \bar{q}_L (i\gamma^\mu D_\mu) q_L + \bar{q}_R (i\gamma^\mu D_\mu) q_R - \bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M}^\dagger q_L .$$

The kinetic terms are invariant in $q_L \to V_L q_L$ and in $q_R \to V_R q_R$. We say that there is $U_L(N_f) \times U_R(N_f)$ symmetry. Both groups are written as $U(N_f) = SU(N_f) \times U(1)$. A linear combination of $U_L(1), U_R(1)$ phases in which both chiralities transform the same way is called $U_V(1)$ (this is the „+1" of p. 39). A linear combination of $U_L(1), U_R(1)$ in which the transformations are opposite is called $U_A(1)$ (this is the transformation of p. 36). In total, the symmetry is

$$\boxed{SU_L(N_f) \times SU_R(N_f) \times U_V(1) \times U_A(1) .}$$

The mass term is invariant under $U_V(1)$, and also under the subgroup $SU_V(N_f)$ (with $V_L \doteq V_R$), provided that $V_L^\dagger \mathcal{M} V_L = \mathcal{M}$, i.e. $\mathcal{M} \propto \mathbb{1}_{N_f \times N_f}$.

Phenomena:



* QCD is believed to explain confinement, i.e. that quarks & gluons always form bound states, hadrons.

* The hadron spectrum and other QCD phenomena are characterized by a confinement scale, of a few hundred MeV. The „light quarks" u,d,s have masses smaller than this; the „heavy quarks" c,b,t are more massive. Therefore, $N_f = 3$ is often a good approximation.

Exercise 9.2:
\# Goldstone bosons
$= \#$ broken generators
$= 2(N_f^2-1) - (N_f^2-1)$
$= N_f^2 - 1$.

* Chiral symmetry is believed to be spontaneously broken even if $\mathcal{M} \to 0$:

$$SU_L(N_f) \times SU_R(N_f) \to SU_V(N_f) .$$

Note that the symmetry breaking pattern is the same as the explicit breaking by $\mathcal{M} \propto \mathbb{1}_{N_f \times N_f}$.

Asymptotic freedom in non-Abelian Yang-Mills theory was first demonstrated by Gross and Wilczek and separately by Politzer in 1973 $\Rightarrow$ Nobel 2004.

* At high energies, QCD is asymptotically free. This means that individual quarks and gluons can be probed in scattering experiments, if the „energy" is much larger than the confinement scale.

A bit more on chiral symmetry: („low-energy QCD")

From Quantum Mechanics:

$$e^{i\alpha\hat{Q}}\,\hat{H}\,e^{-i\alpha\hat{Q}} = \hat{H} \qquad \text{(invariance)}$$

$$\Leftrightarrow \quad [\hat{Q},\hat{H}] = \hat{0} \qquad \text{(commuting observables)}$$

$\Rightarrow$ eigenstates of $\hat{Q}$ are also eigenstates of $\hat{H}$

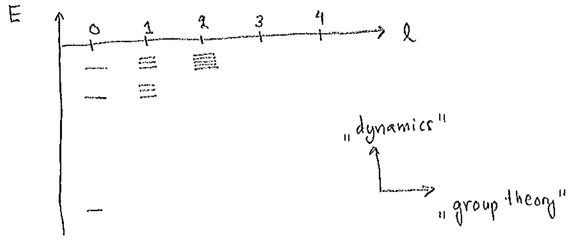
$\Rightarrow$ degeneracies of eigenstates depend on representations of symmetry group.

Example: Hydrogen atom

Symmetry group: Rotations, SO(3)

$\Rightarrow$ $(2l+1)$-dimensional representations, $l = 0,1,2,\ldots$

$\Rightarrow$ degeneracy $2l+1$.

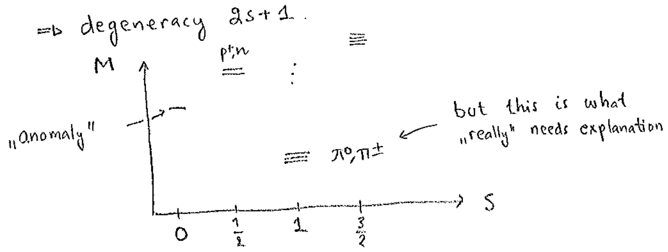


Analogously with hadron spectra:

(i) Symmetry group: $SU_V(N_f=2)$ „isospin" $\binom{u}{d}$

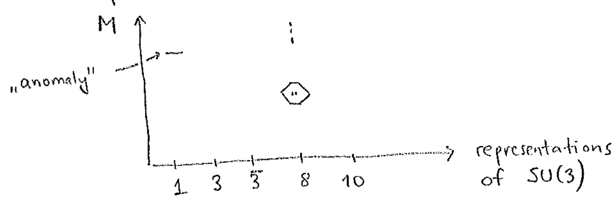
$\Rightarrow$ $(2s+1)$-dimensional representation, $s = 0,\frac{1}{2},1,\ldots$

$\Rightarrow$ degeneracy $2s+1$.

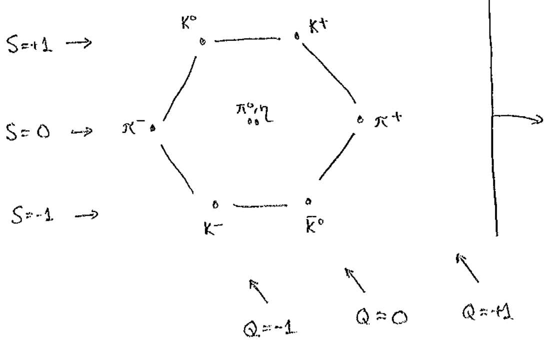


(ii) Symmetry group $SU_V(N_f=3)$, including s

$\Rightarrow$ representations have degeneracies $1,3,8,\ldots$



Gell-Mann's „eightfold way" for the lightest mesons:

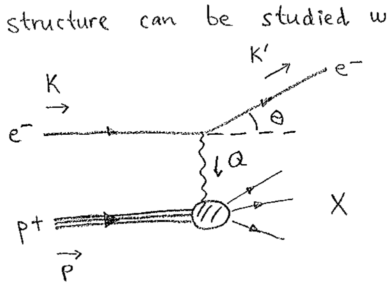


Remarks:

* The (approximate) degeneracies of hadron spectra let us deduce something about the underlying global symmetry.
* The dynamical oddity that non-trivial representations are the lightest requires a more subtle explanation. We believe that π's & K's are (approximate) Goldstone bosons of the symmetry breaking $SU_L(N_f)\times SU_R(N_f) \to SU_V(N_f)$.
* Symmetry manifests itself not only in spectra but also in scattering amplitudes, cf. Exercise 11.2.

A bit more on asymptotic freedom: („high-energy QCD")

Hadron structure can be studied with „Deep Inelastic Scattering" (DIS).



(One of the possible follow-ups of LHC is „LHeC".)

← an „inclusive" final state: only the outgoing electron is identified.

Known: K, P Measured: E', θ $\Leftrightarrow$ $K' = (E', \sqrt{E'^2 - m_e^2}\,\vec{e}_{k'})$.

Construct:
$$\begin{cases} Q_E^2 := -Q^2 & \text{„virtuality"} \\ x := \dfrac{Q_E^2}{2Q\cdot P} & \text{„Bjorken } x\text{"} \end{cases}.$$

For $E \gg m_e$ the cross section can be parametrized as

$$\frac{d\sigma}{dE'\,d\Omega} =: \left(\frac{\alpha_{em}}{2E\sin^2(\theta/2)}\right)^2 \Big[\, 2W_1(Q_E^2,x)\sin^2\!\left(\tfrac{\theta}{2}\right) + W_2(Q_E^2,x)\cos^2\!\left(\tfrac{\theta}{2}\right) \Big].$$

„structure functions"

Peculiarities: $*$ For $Q_E^2 \gtrsim (1\text{ GeV})^2$ everything depends on one variable only:

$$m_p W_1(Q_E^2,x) \to F_1(x),$$
$$\frac{Q_E^2}{2m_p x} W_2(Q_E^2,x) \to F_2(x).$$ „Bjorken scaling" (1969)

$*$ Moreover, if x is not too small, there is only one function:

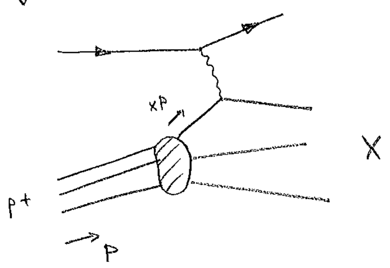
$$2x\,F_1(x) = F_2(x)$$ „Callan-Gross relation" (1969)

In other words, the complicated proton has a hidden simple structure for high virtualities.

Interpretation: („parton model")

Bjorken et al; Feynman 1972

For high virtualities we see asymptotically free „partons":



There is a probability, called a „parton distribution function" (PDF), f_i, with which we can take out a parton with momentum xP, and let it scatter elastically on the photon.

Total momentum: $P = \int_0^1 dx \sum_i f_i(x)\, x P$.

Valence quarks (p. 42): $f_i(x) \to u_v(x), d_v(x)$.

Sea quarks (p. 42): $f_i(x) \to s(x), \bar{s}(x)$.

Gluons: $f_i(x) \to g(x)$.

For small x the partons are „soft" and the „free" description breaks down.