3.4. Glashow - Weinberg - Salam model

The problems of the Fermi model (sec. 3.3) can be solved in a relatively simple* way through a model formulated around 1967. [That this is the case became clear only in 1971 when 't Hooft and Veltman proved the model's renormalizability.]

The essential ingredient is gauge invariance, so let us recall what this means.

Abelian case (p. 5):

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}_e (i\gamma^\mu D_\mu - m_e) \Psi_e + \ldots + \mathcal{L}_{g.f.} \;;$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \;; \qquad D_\mu = \partial_\mu - ieA_\mu .$$

Gauge transformation:
$$\begin{cases} \Psi_e \to \Psi_e' = e^{ie\Theta(x)} \Psi_e \\ \bar{\Psi}_e \to \bar{\Psi}_e' = \bar{\Psi}_e e^{-ie\Theta(x)} \\ A_\mu \to A_\mu' = A_\mu + \partial_\mu \Theta(x) \end{cases}$$

$$\Rightarrow D_\mu' \Psi_e' = \{\partial_\mu - ieA_\mu'\} e^{ie\Theta(x)} \Psi_e$$
$$= e^{ie\Theta(x)} \{\partial_\mu \Psi_e + ie\cancel{\partial_\mu \Theta}\, \Psi_e - ieA_\mu \Psi_e - ie\cancel{\partial_\mu \Theta}\, \Psi_e\}$$
$$= e^{ie\Theta(x)} D_\mu \Psi_e .$$

Therefore, $\bar{\Psi}_e' (i\gamma^\mu D_\mu' - m_e) \Psi_e' = \bar{\Psi}_e (i\gamma^\mu D_\mu - m_e) \Psi_e$.

Implications:

* Since $W^\pm, Z^0$ are massive vector particles (cf. p. 22),
$$\Delta^{-1}_{\mu\nu} = \frac{-i\eta_{\mu\nu}}{p^2 - M^2} ,$$
the Lagrangian should contain (cf. Exercise 2.1)
$$\mathcal{L} = \frac{1}{2} A^\mu (\partial^\alpha \partial_\alpha + M^2) \eta_{\mu\nu} A^\nu + \ldots$$
Unfortunately the new term is not gauge invariant:
$$\frac{1}{2} M^2 A'^\mu A_\mu' = \frac{1}{2} M^2 \{A^\mu A_\mu + \partial^\mu \Theta A_\mu + A^\mu \partial_\mu \Theta + (\partial_\mu \Theta)^2\}.$$

* There is a similar problem with fermions because, unlike in QED, in weak interactions left- and right-handed fields transform differently:
$$\Psi_L \to \Psi_L' = e^{ie\Theta(x)} \Psi_L ,$$
$$\Psi_R \to \Psi_R' = e^{ie\chi(x)} \Psi_R .$$
A gauge-invariant mass term would be $m\bar{\Psi}_L \Psi_L$ but this vanishes:
$$\bar{\Psi}_L \Psi_L = \Psi^\dagger P_L \gamma^0 P_L \Psi = \bar{\Psi} P_R P_L \Psi \underset{\text{Exercise 4.3}}{=} 0 .$$

* Only scalar particles can trivially be made massive:
$$\phi \to \phi' = e^{ie\Theta} \phi$$
$$\phi^\dagger \to \phi'^\dagger = \phi^\dagger e^{-ie\Theta}$$
$$\Rightarrow \delta\mathcal{L} = m^2 \phi^\dagger \phi \quad \text{is invariant.}$$

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* But surely not unique!

Basic philosophy:

* Introduce a scalar field (spin-0), which is given a mass.
* Let the scalar interact with fermions (spin-$\frac{1}{2}$) and vectors (spin-1). Interactions should „transmit" the mass to them.
* To guarantee renormalizibility, interactions should be gauge invariant, and only vertices with couplings of non-negative dimensionality are allowed (cf. p. 24).
* Given these principles, write down the most general Lagrangian possible, and fix the parameters appearing through a comparison with experiment.

Field content:

* Spin-0: Higgs field Φ.
* Spin-$\frac{1}{2}$: Three generations of chiral fermions*:

$$L_{1L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad L_{2L} = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \qquad L_{3L} = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$(\nu_e)_R \qquad (\nu_\mu)_R \qquad (\nu_\tau)_R$$

$$e_R \qquad \mu_R \qquad \tau_R$$

$$Q_{1L} = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad Q_{2L} = \begin{pmatrix} c \\ s \end{pmatrix}_L \qquad Q_{3L} = \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$u_R \qquad c_R \qquad t_R$$

$$d_R \qquad s_R \qquad b_R$$

* Spin-1: Vector bosons related to the gauge symmetry:

$$\underset{\substack{\uparrow \\ \text{„hypercharge"} \\ \left(\text{electromagnetic interaction}\right)}}{U_Y(1)} \times \underset{\substack{\uparrow \\ \text{„left-handed"} \\ \left(\text{weak interaction}\right)}}{SU_L(2)} \times \underset{\substack{\uparrow \\ \text{„colour"} \\ \left(\text{strong interaction}\right)}}{SU_c(3)}$$

* Each field is assigned „quantum numbers", specifying how it changes under the various gauge groups.

* We have changed the notation from p. 19, leaving out primes and interpreting the fields as weak interaction eigenstates. But note that this convention is not universal.

Remarks:

* The „elegance" of the Glashow-Weinberg-Salam model lies in a relatively simple and „minimal" basic philosophy.
* However the practical realization is complicated, because of the request for the „most general" Lagrangian.
* Nevertheless nature does appear to make use of the full generality!
* A deeper „explanation" for why there are three generations, why the quantum numbers are what they are, etc, is „Beyond the Standard Model". (But see also: „Weak Anthropic Principle".)

General structure:

$$\mathcal{L}_{MSM} = (\text{QED-type terms}) + (\text{terms involving } \Phi)$$

„Minimal Standard Model"

(i) „kinetic term" $\sim (D_\mu\Phi)^\dagger(D^\mu\Phi)$
$\Rightarrow$ masses for $W^\pm, Z^0$

(ii) „self-interactions" $\sim -V(\Phi^\dagger\Phi)$
$\Rightarrow$ (Anderson)-Brout-Englert-Higgs-(Kibble) mechanism [1962-64]

(iii) „Yukawa terms" $\sim \bar{q}_R \Phi^\dagger Q_L$
$\Rightarrow$ masses for fermions

Dimensionalities:

$$\underbrace{S_{MSM}}_{\text{GeV}^0} = \int \underbrace{d^4x}_{\text{GeV}^{-4}} \; \underbrace{\mathcal{L}_{MSM}}_{\text{GeV}^4} \quad ; \quad \underbrace{\mathcal{L}_{MSM}}_{\text{GeV}^4} \sim \bar{Q} \underbrace{\not{\partial}}_{\text{GeV}} Q + (\underbrace{D_\mu}_{\text{GeV}}\Phi)^\dagger (\underbrace{D^\mu}_{\text{GeV}}\Phi)$$

$$\Rightarrow [Q] = [\bar{Q}] = \text{GeV}^{3/2}$$
$$[\Phi] = [\Phi^\dagger] = \text{GeV}$$

So, the terms $[\Phi^\dagger\Phi] = \text{GeV}^2$, $[(\Phi^\dagger\Phi)^2] = \text{GeV}^4$, $[\bar{q}_R\Phi^\dagger Q_L] \sim \text{GeV}^4$ can be multiplied by couplings of non-negative dimension; everything else, e.g. $[(\Phi^\dagger\Phi)^3] = \text{GeV}^6$, $[\bar{q}_R\Phi^\dagger Q_L \Phi^\dagger\Phi] = \text{GeV}^6$ would require a coupling of negative dimensionality and is excluded by renormalizability.

Detailed structures:

(i) Higgs kinetic term

The Higgs field is chosen to carry the following quantum numbers:

* neutral under $SU_c(3)$
* transforms under $SU_L(2)$
$\Rightarrow \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$
* transforms under $U_Y(1)$ with „charge" $Y = -\frac{1}{2}$.

The covariant derivative has the form

$$D_\mu = \partial_\mu - i g_w T^a A^a_\mu + \frac{1}{2} i g_Y B_\mu$$

$$= \partial_\mu - i g_w T^a A^a_\mu + i g_Y T^0 B_\mu$$

weak coupling (p. 22) (g_w)

2×2-matrices: $T^a := \frac{\sigma^a}{2}$

3 gauge fields $\to W^\pm, Z^0$

1 gauge field $\to \gamma$

$T^0 := \frac{\mathbb{1}_{2\times 2}}{2}$

hypercharge coupling (g_Y)

hypercharge $-\frac{1}{2}$

The corresponding term in the Lagrangian is

$$\delta\mathcal{L} = (D_\mu\Phi)^\dagger (D^\mu\Phi) .$$

Consequences $\Rightarrow$ sec. 3.5.

(ii) Higgs self-interactions

The terms only containing the Higgs field are called the „Higgs potential":

$$\delta\mathcal{L} = -V(\Phi^\dagger\Phi).$$

As discussed on p. 27, only two gauge-invariant combinations can appear:

$$V(\Phi^\dagger\Phi) = \mu^2\,\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2,$$

where μ^2, λ are free parameters $\Rightarrow$ sec. 3.6.

(iii) Yukawa terms

It turns out that the Higgs field can appear in two different forms (cf. Exercise 7.2):

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix};$$

$$\tilde{\Phi} := i\sigma^2\Phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \phi^{+*} \\ \phi^{0*} \end{pmatrix} = \begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix}.$$

The possible structures are (cf. Exercise 7.3):

$$\begin{aligned} \delta\mathcal{L} = &- \left[h_u^{i1}\,\bar{Q}_{iL}\,\tilde{\Phi}\,u_R + \text{H.c.} \right] \\ &- \left[h_d^{i1}\,\bar{Q}_{iL}\,\Phi\,d_R + \text{H.c.} \right] \\ &+ (\text{2nd \& 3rd generation}) + (\text{leptons}). \end{aligned}$$

Here one must be careful to allow for the most general structure; but at the same time, no redundancies should be introduced either.

We return to this in sec. 3.7.

(iv) QED-type terms

These include generalizations of $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ for $U_Y(1)$, $SU_L(2)$ and $SU_c(3)$ gauge bosons; as well as „kinetic terms" for all fermions:

$$\delta\mathcal{L} = \sum_{k=1}^{3} \bar{L}_k\, i\gamma^\mu D_\mu L_k + (\text{quarks});$$

$$\begin{aligned} D_\mu = &\ \partial_\mu - i\left[g_w T^a A_\mu^a + g_Y T^0 B_\mu\right]P_L \\ &- i\left[g_Y \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} B_\mu\right]P_R \end{aligned}$$

hypercharge matrix for right-handed leptons.

Hypercharges for quarks $\Rightarrow$ Exercise 7.3.