3.2 Fermi model

Having already anticipated that weak interactions break many symmetries, we now start „building up" the corresponding theory. We follow a roughly historical line, making only minimal improvements in every step.

(i) Fermi model [1932]

After the experimental observation that energy does not appear to be conserved in the β-decay of neutrons, Pauli introduced the neutrino:

$$n \to p^+ + e^- + \bar{\nu}_e \,.$$

To be quantitative, Fermi proposed a one-parameter model for this process, making use of QED-type structures:

$$\mathcal{L}_I = -G_F \left(\bar{n}\gamma^\mu p \; \bar{\nu}_e \gamma_\mu e + \bar{p}\gamma^\mu n \; \bar{e}\gamma_\mu \nu_e \right).$$

real parameter, $G_F \approx 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$

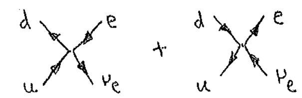
Lorentz-symmetric

Hermitean conjugate

Today we would replace nucleons with quarks:

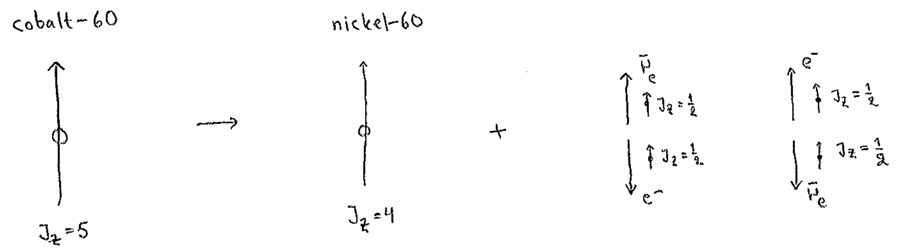
$$\mathcal{L}_I = -G_F \left(\bar{d}\gamma^\mu u \; \bar{\nu}_e \gamma_\mu e + \text{H.c.} \right)$$

Vertices:



(ii) Parity violation [1957]

In the mid-1950's the question was posed whether parity is always conserved. Lee and Yang searched literature and found that many experiments confirmed the absence of parity violation in strong decays, but none in weak decays [1956]. C.S. Wu carried subsequently out a new experiment [1957]:



observed!   not observed!

Interpretation: the massless $\bar{\nu}_e$ has helicity $+\frac{1}{2}$

$\Downarrow$

the massless ν_e has helicity $-\frac{1}{2}$

$\Downarrow$ ← Exercise 5.1

the ν_e-field is left-handed

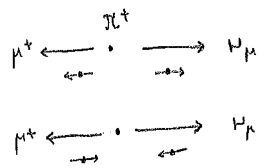
$\Downarrow$ ← p. 14

parity is violated!

A slightly simpler experiment [Lederman et al 1957]:

$$\pi^+ \longrightarrow \mu^+ \nu_\mu$$

In the rest frame of the decaying pion:



not observed

observed: „muons are strongly polarized" (neutrinos are not seen)

(Incidentally, why not $\pi^+ \to e^+ \nu_e$? Given that positron is „almost massless", $m_e / m_\pi \approx 0.5/140$, it behaves like an antineutrino: $e^+ \longleftarrow \cdot \longrightarrow \nu_e$ ↯)

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These observations necessitate an improvement of the Fermi model. From Exercises 4.3, 5.1 it follows that the desired „left chirality" of massless fields can be achieved, without breaking invariance under <u>proper</u> Lorentz transformations, through the replacement

$$\psi \to \psi_L := P_L \psi = \frac{\mathbb{1} - \gamma^5}{2} \psi .$$

The combination $\bar\psi_i \gamma^\mu \psi_j$ is called a „vector current", the combination $\bar\psi_i \gamma^\mu \gamma^5 \psi_j$ an „axial current" (cf. p. 13).

So, the new model is called the „<u>V-A" - model</u>:

$$\mathcal{L}_I^{V-A} = -2\sqrt{2}\, G_F \left(\bar d_L \gamma^\mu u_L \, \bar\nu_{eL} \gamma_\mu e_L + \text{H.c.} \right)$$

So chosen that „old" predictions like $n \to p^+ + e^- + \bar\nu_e$ remain intact; requires an explicit computation of $|\mathcal{M}|^2$.

$$= -2\sqrt{2}\, G_F \left(\bar d \gamma^\mu P_L u \, \bar\nu_e \gamma_\mu P_L e + \text{H.c.} \right) .$$

$$\psi^\dagger P_L \gamma^0 \gamma^\mu P_L = \psi^\dagger \gamma^0 P_R \gamma^\mu P_L$$
$$= \psi^\dagger \gamma^0 \gamma^\mu P_L P_L = \bar\psi \gamma^\mu P_L$$

(To be precise, things become somewhat delicate once the theory is „regularized" through dimensional regularization: if $\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3$, then $\bar d_L \gamma^\mu u_L = \bar d \gamma^\mu P_L u$ only for $\mu = 0,1,2,3$, whereas $P_R \gamma^x P_L = \gamma^x + \gamma^5\gamma^x - \gamma^x\gamma^5 - \gamma^5\gamma^x\gamma^5 = 0$ for $x \notin \{0,1,2,3\}$, because then $[\gamma^5, \gamma^x] = 0$.)

(iii) Non-conservation of particle / flavour numbers [... 1963]

Continuous symmetries (p. 16) are also violated in weak interactions. Examples:

* $n = udd \to p^+ = uud + e^- + \bar{\nu}_e$ $\Rightarrow$ particle numbers and isospin are not conserved
  $I_z = +\frac{1}{2}-\frac{1}{2}-\frac{1}{2}$ $\quad$ $I_z = +\frac{1}{2}+\frac{1}{2}-\frac{1}{2}$
* $K^+ = u\bar{s} \to \pi^+\pi^0\,,\ \pi^+\pi^0\pi^0$ $\Rightarrow$ „strangeness" is violated
* $D^0 = c\bar{u} \to K^-\pi^+\,,\ K^-\pi^+\pi^0$ $\Rightarrow$ „charmness" is violated

In QED (or QCD) this would not happen, since the structure is always $\bar{\Psi}\gamma^\mu\Psi$. Some of these reactions were already included in $\mathcal{L}_I^{V-A}$ but not quite correctly, so a further generalization is needed.

Denote:

$$L_1 := \begin{pmatrix} \nu_e \\ e \end{pmatrix} \ ; \quad L_2 := \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{matrix} \leftarrow Q = 0 \\ \leftarrow Q = -1 \end{matrix}$$

$$Q_1 := \begin{pmatrix} u \\ d \end{pmatrix} \ ; \quad Q_2 := \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{matrix} \leftarrow Q = \frac{2}{3} \\ \leftarrow Q = -\frac{1}{3} \end{matrix}$$

Q = electric charge in units of e.

Question: Since the „quantum numbers" of ν_e and ν_μ; e and μ; u and c; d and s are identical, how can we keep them apart?

Answer: We cannot! We can define L_1 to be the doublet with e, L_2 with μ, Q_1 with u, Q_2 with c. But then we should replace d and s with an arbitrary linear combination:

$$Q_1' := \begin{pmatrix} u \\ d' \end{pmatrix} \ ; \quad Q_2' := \begin{pmatrix} c \\ s' \end{pmatrix} \ ;$$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \ ;$$

θ_c = „Cabibbo-angle" $\approx 13.1°$ [1963]

Remarks:

* Should one not do the same with ν_e and ν_μ? In principle yes, but since both are almost massless, we can usually treat them as identical, and then a rotation has no effect.
* Of course, ultimately we will have to generalize the structure to three generations (sec. 3.7).

Summary:

$$\mathcal{L}_I^{V-A} \overset{p.18}{=} -2\sqrt{2}\, G_F \left\{ \bar{Q}_{1L} \begin{pmatrix} 0 & 0 \\ \gamma^\mu & 0 \end{pmatrix} Q_{1L}\, \bar{L}_{1L} \begin{pmatrix} 0 & \gamma_\mu \\ 0 & 0 \end{pmatrix} L_{1L} + \text{H.c.} \right\}$$

$$\overset{p.19}{\Rightarrow} -2\sqrt{2}\, G_F \sum_{D_L, D_L'} \bar{D}_L \begin{pmatrix} 0 & 0 \\ \gamma^\mu & 0 \end{pmatrix} D_L\, \bar{D}_L' \begin{pmatrix} 0 & \gamma_\mu \\ 0 & 0 \end{pmatrix} D_L' ,$$

where $D_L, D_L' \in \{ Q_{1L}', Q_{2L}', L_{1L}, L_{2L} \}$.

A simple structure but contains lots of physics!

Example 1: $\boxed{\mu^- \to e^- \bar{\nu}_e \nu_\mu}$

Choose $D_L = L_{1L}$, $D_L' = L_{2L}$, or vice versa.

$$\Rightarrow \mathcal{L}_I^{V-A} = -2\sqrt{2}\, G_F \left\{ \bar{e}_L \gamma^\alpha \nu_{eL}\, \bar{\nu}_{\mu L} \gamma_\alpha \mu_L + \text{H.c.} \right\}.$$

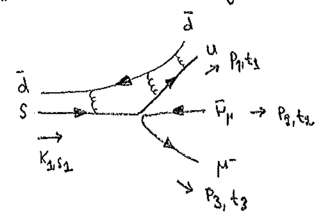
Example 2: $\boxed{\bar{K}^0 \to \pi^+ \mu^- \bar{\nu}_\mu}$

hadrons   leptons

$\bar{K}^0 = s\bar{d}$; $\pi^+ = u\bar{d}$

„semileptonic decay"

Feynman diagram:



Choose $D_L = Q_{1L}'$, $D_L' = L_{2L}$, or vice versa.

$$\Rightarrow \mathcal{L}_I^{V-A} = -2\sqrt{2}\, G_F \sin\theta_c \left\{ \bar{s}_L \gamma^\alpha u_L\, \bar{\nu}_{\mu L} \gamma_\alpha \mu_L + \text{H.c.} \right\}.$$

For the amplitude one gets ($\eta := i^n$; $n \in \mathbb{Z}$)

$$\mathcal{M} = \eta \cdot \left(\frac{-G_F}{\sqrt{2}} \right) \cdot \sin\theta_c \left\{ \bar{u}_u(\vec{p}_1, t_1)\, \gamma^\alpha (1-\gamma^5)\, u_s(\vec{k}_1, s_1) \times \bar{u}_\mu(\vec{p}_3, t_3)\, \gamma_\alpha (1-\gamma^5)\, v_{\nu_\mu}(\vec{p}_2, t_2) \right\}.$$

But actually, given that the hadronic part involves strong interactions, it would be better not to use free spinors there. One could rather write, with a proper normalization of the states and re-introducing quantum-mechanical operators:

$$\mathcal{M} = \eta \left(-\frac{G_F}{\sqrt{2}} \right) \sin\theta_c \left\{ \langle \pi^+(\vec{p}_1, t_1) |\, \hat{u}\, \gamma^\alpha (1-\gamma^5)\, \hat{s}\, | \bar{K}^0(\vec{k}_1, s_1) \rangle \times \bar{u}_\mu(\vec{p}_3, t_3)\, \gamma_\alpha (1-\gamma^5)\, v_{\nu_\mu}(\vec{p}_2, t_2) \right\}.$$

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