Standard Model

(Mikko Laine ; ExWi-117)

* Course material (script, exercises) through ILIAS

* Schedule:

Exercises	Mon	10-12	B1
Lectures	Mon	14-15	119
	Wed	13-14	119

  There may be a couple of weeks where exercises and lectures swap places.

* Literature: in the „semester depot"

* Scope: lectures $\leqslant$ script $<$ literature

  $\downarrow$ exercises

  $\downarrow$ exam

* Contents:
  1. Introduction
  2. QED
  3. $\boxed{\text{3. Weak interactions}}$
  4. QCD
  5. Beyond the Standard Model (BSM)

* Please note: This semester there is a Specialist Course called „Introduction to BSM physics" (Mon 15-17 119, from 25.2.). It should complement the current lecture very nicely, so please listen to it as well!

* Exam: Later on, a list of about 10 „themes" is put online. In the oral exam, 2 of the themes are chosen randomly. Each theme is discussed for about 20 minutes.

  Time slots are available in the 1st and 3rd June week.

* Questions: Most welcome!

1. Introduction

Basic concepts:

* **Fermions**
  - Leptons: $\begin{pmatrix}\nu_e\\ e\end{pmatrix}\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}\begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}$
  - Quarks: $\underbrace{\begin{pmatrix}u\\ d\end{pmatrix}\begin{pmatrix}c\\ s\end{pmatrix}\begin{pmatrix}t\\ b\end{pmatrix}}_{\text{Generation}}$ $\updownarrow$ Isospin $\rightsquigarrow$ Hadrons — Baryons $(p, n, \dots)$; Mesons $(\pi, K, \dots)$

* **Bosons**:

Photon	γ	„ $U(1)$"
Vektor bosons	$W^\pm, Z^0$	„ $SU_L(2)$"
Gluons	g	„ $SU(3)$"
Higgs	H	

* Antiparticles are denoted by a bar, e.g. antiquark $\bar{q}$; $q \in \underbrace{\{u,d,s,c,b,t\}}_{\text{„Flavour"}}$

* Apart from the particle content listed above, the theory is characterized by a number of parameters: masses (m_e, m_μ, m_τ; $m_u, m_d, m_s, \dots$; $M_{W^\pm}, M_{Z^0}, M_H$) as well as coupling constants ($e, g_w, g_s, \dots$). These are only known with a finite precision; the current best values are summarized by the „Particle Data Group" [pdg.lbl.gov].

* We will be using „natural units", referred to as „$\hbar = c = k_B = 1$". Here $\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$ Js ; $c = 299792458 \frac{m}{s}$; $k_B = 1.381 \times 10^{-23} \frac{J}{K}$.
  The rule is that any quantity is multiplied by powers of $\hbar, c, k_B$ until it has the dimension of energy; and energy is subsequently measured in units of $\text{GeV} = 10^9\, \text{eV} = 10^9 \cdot 1.602 \times 10^{-19}\, \text{C} \cdot \text{V} = 1.602 \times 10^{-10}\, \text{J}$.

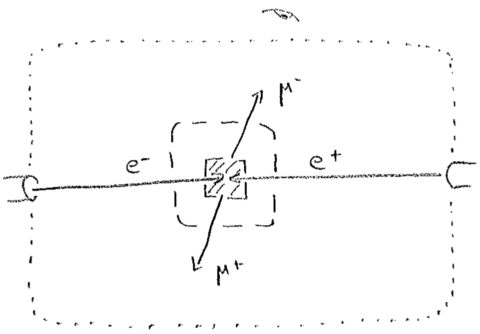
  (Example: $\boxed{\text{GeV} \times \text{fm} \approx 5}$

  Proof: $$\text{GeV} \times 10^{-15}\, \text{m} := \text{GeV} \times 10^{-15}\, \text{m} \times \frac{1}{3.0 \times 10^8 \frac{m}{s}} \times \frac{1}{1.1 \times 10^{-34}\, \text{Js}}$$
  $$= \frac{1.6 \times 10^{-10} \times 10^{-15}}{3.3 \times 10^8 \times 10^{-34}} \approx 0.5 \times 10^1 \quad \square .$$)

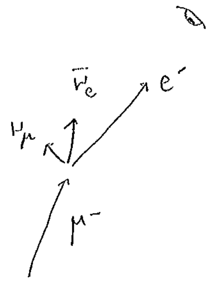
* It is important to be fluent in relativistic kinematics.
  We use the metric convention $\eta^{\mu\nu} = \eta_{\mu\nu} = \text{diag}\,(+\,-\,-\,-)$.
  So, $$p^2 = p \cdot p = p^\mu p_\mu = (p^0)^2 - \vec{p}^{\,2} = E^2 - \vec{p}^{\,2} = \underbrace{(m^2 + \vec{p}^{\,2})}_{\text{Einstein}} - \vec{p}^{\,2} = m^2 .$$
  $p = (p^0, \vec{p})$

  (In general units: $p = \left(\frac{E}{c}, \vec{p}\right)$; $E = \sqrt{(mc^2)^2 + (\vec{p}c)^2}$)

The big picture (sketch)

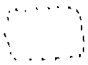
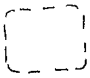


Scattering

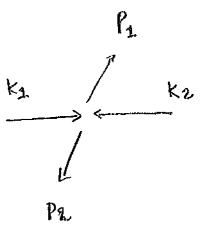


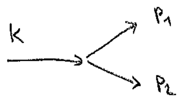
Decay

Understanding these processes involves three different „levels":

	level	what is going on	nature of physics; $\boxed{\text{keyword}}$
(i)	experiment 	measure rates $\left(\frac{\text{events}}{\text{time}}\right)$; divide outcoming by ingoing in order to get probability, expressed as „cross section"	Classical relativistic kinematics; $\boxed{\text{phase space}}$
(ii)	intermediate 	relativistic particles propagate towards or away from interaction region, „touching it".	relativistic „one-particle" quantum mechanics; $\boxed{\text{invariant amplitude}}$
(iii)	interactions 	laws of Standard Model cause transitions.	relativistic „many-particle" quantum field theory; „amputated $\boxed{\text{Green's functions}}$"

Link between (i) and (ii) : $\boxed{\text{Fermi's Golden Rule}}$ (cf. time-dependent phenomena; Dirac picture)





For scattering : $d\sigma = \frac{1}{F}\, d\Phi_n\, |\mathcal{M}|^2$

For decays : $d\Gamma = \frac{1}{2E_k}\, d\Phi_n\, |\mathcal{M}|^2$

Here σ = „cross section"; Γ = „decay rate" ;

$\mathcal{M}$ = „invariant amplitude"

F = „flux factor" $= 4\sqrt{(k_1\cdot k_2)^2 - m_1^2 m_2^2}$

$d\Phi_n$ = „phase space" $= \prod_{i=1}^{n} \frac{d^3\vec p_i}{(2\pi)^3 2E_{p_i}}\, (2\pi)^4 \delta^{(4)}\!\left(\sum_j k_j - \sum_i p_i\right)$.

Link between (ii) and (iii) : $\boxed{\text{LSZ -reduction}}$ (cf. QFT II)

Lehmann- Symanzik- Zimmermann

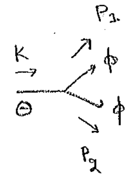
$\mathcal{M} \sim (\text{free spinors})^\dagger\,(\text{amputated Green's})\,(\text{free spinors})$.

Example :

$$\mathcal{L}_I := \underbrace{\tfrac{1}{2}\, g}_{\text{coupling}}\; \underbrace{\Theta\, \phi^2}_{\text{scalar fields}}$$

$\mathcal{L}_I$: interaction

Let Θ and ϕ have masses M and m, and let $M \geq 2m$.
What is the decay rate (or „inverse life time") Γ_Θ ?



__Note on identical particles__ :

It is customary to define couplings so that there is $\frac{1}{n!}$ in the Lagrangian for identical fields. This gets cancelled when computing amputated Green's functions through Wick's theorem:

$$\langle \Theta(K)\, \phi(p_1)\, \phi(p_2) \quad \tfrac{1}{2}\, g\, \Theta(x)\, \phi(x)\, \phi(x) \rangle_0$$

$$= g\; \phi(p_1)\phi(x)\;\; \phi(p_2)\phi(x)\;\; \Theta(K)\Theta(x)$$

expectation value with respect to free theory

However if one integrates over full phase space, a division by $\frac{1}{n!}$ needs to be re-introduced in Γ (or σ), to avoid double-counting of identical configurations.

__Computation__ :

Choose a frame in which Θ is at rest: $K = (M, \vec{0})$.
For scalar particles spinors are trivial, so $|\mathcal{M}| = g$.

Thereby

$$\Gamma = \frac{1}{2} \cdot \frac{1}{2M} \cdot \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_{p_1}} \int \frac{d^3\vec{p}_2}{(2\pi)^3 2E_{p_2}} \cdot (2\pi)^4 \delta^{(4)}(K - P_1 - P_2)\, |g|^2$$

identical particles in the final state

$$= \frac{|g|^2}{16M} \cdot \frac{1}{(2\pi)^2} \cdot \int \frac{d^3\vec{p}_1}{E_{p_1}} \int \frac{d^3\vec{p}_2}{E_{p_2}} \cdot \delta^{(3)}(-\vec{p}_1 - \vec{p}_2)\, \delta(M - E_{p_1} - E_{p_2})$$

$\vec{p}_2 = -\vec{p}_1$

$$\overset{\uparrow}{=} \frac{|g|^2}{16M} \cdot \frac{1}{(2\pi)^2} \cdot \int \frac{d^3\vec{p}_1}{E_{p_1}^2}\; \delta(M - 2E_{p_1})$$

$p_1 := |\vec{p}_1|$

$$\overset{\uparrow}{=} \frac{|g|^2}{16M} \cdot \frac{1}{(2\pi)^2} \cdot 4\pi \cdot \int_0^\infty \frac{dp_1\, p_1^2}{p_1^2 + m^2}\; \delta\left(M - 2\sqrt{p_1^2 + m^2}\right)$$

$$= \frac{|g|^2}{16\pi M} \cdot \Theta(M - 2m) \cdot \left\{ \frac{p_1^2}{p_1^2 + m^2} \cdot \frac{1}{2 \cdot \frac{d}{dp_1}\sqrt{p_1^2 + m^2}} \right\}_{M = 2\sqrt{p_1^2 + m^2}}$$

$$= \frac{|g|^2}{16\pi M} \cdot \Theta(M - 2m) \cdot \left\{ \frac{p_1}{2\sqrt{p_1^2 + m^2}} \right\}_{\sqrt{p_1^2 + m^2} = \frac{M}{2}\,;\; p_1 = \frac{\sqrt{M^2 - 4m^2}}{2}}$$

$$= \frac{|g|^2}{32\pi M}\, \Theta(M - 2m) \cdot \sqrt{1 - \frac{4m^2}{M^2}}$$

__Analogous physical processes__ : $H \to Z^0 Z^0$; $\pi^0 \to \gamma\gamma$; ...

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